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Resistivity Measurements of Conductors and Semiconductors of Different Geometrical Shapes Using Van der Pauw Technique

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Abstract— The resistivity of conductive and semi-conductive materials, are determined using an algorithmic computation approach of Van der Pauw technique. Different geometrical shapes, such as bulk, surface, bars, wafers, are used are implemented. The four-point method has been extended to multi-point measurement of surface and volume geometrical shapes, with consequently rearranging the equations derived in Van der Pauw technique. It is proposed that the multi-point measurements derivations can be applied using a multiplexer (MUX) and a de-multiplexer (de-MUX) digital circuits, and further on, embedded microprocessors and microcontrollers.

Index Terms— resistivity, Van der Pauw, conductors, semiconductors, shapes, geometries, equation derivations, digital, MUX, de-MUX

1 INTRODUCTION

To measure the resistivity of low resistance materials such as contacts, foils, bars rods and superconductors, very sensitive voltmeter with a current source or a micro-ohmmeter are required. The measurement is subject to additional erroneous sources including lead resistance, non-ohmic contacts, thermoelectric EMFs and device heating.

Semiconductors, on the other hand, have high resistance and can induce significant noise sources such as Johnson noise. It must also be noted that high resistance measurements are subject to a loading errors from the meter input impedance, as well as the impedance of the connecting cable. This requires extra efforts to reduce errors such as the use of very high input resistance meters as well as guarding cables and wires used.

Other noise sources are magnetic fields and ground loops arising from leakage currents due to insulated materials in the circuit measurement. In order to measure voltages from high-resistance sources accurately, the insulation leakage resistance of the test fixtures, test leads, and measuring voltmeter must be several orders of magnitude higher than the Thevenin equivalent resistance of the circuit under test, depending on the accuracy and resolution required. This is due to the shunting effects of insulators, which makes detecting inferior insulation in test-ups difficult.

Temperature and humidity effects on noise and insulation can also generate erroneous measurements. High temperature can increase Johnson voltage noise, whereas high humidity can degrade the function of insulation

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2 RESISTIVITY MEASUREMENTS

2.1 The Four-Point Method

Due to the limitations of a typical two-wire method in which a test current is forced through the test leads and the resistance being measured, and the voltage drop across sample resistance is measured; the 4-wire (Kelvin) connection method [1], [2], [3], [4], is generally preferred, as depicted in **Fig. 1**. It can be seen that some small current may flow through the sense leads, but generally can be ignored for smaller length leads. The resistivity is calculated as follows:

$$\rho = k\,V / I \qquad (1)$$

where $\boldsymbol{V}$ = the measured voltage, I = the source current and k = sample material and shape constants.

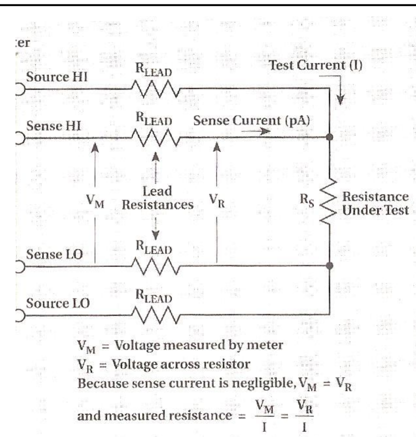

Fig. 1. The four-point method is used for rmeasuring resistance.

2.2 Van der Pauw Technique

Van der Pauw technique [5], [6], implements the four-wire method to reduce thermoelectric EMFs effect. It is particularly useful for measuring very small samples because the dimensions of the sample and the spacing of the contacts are unimportant. This technique uses four isolated contacts on the boundary of a flat, arbitrary shaped sample. A total of eight measurements, are made around the sample, [7], as illustrated in **Fig. 2.**

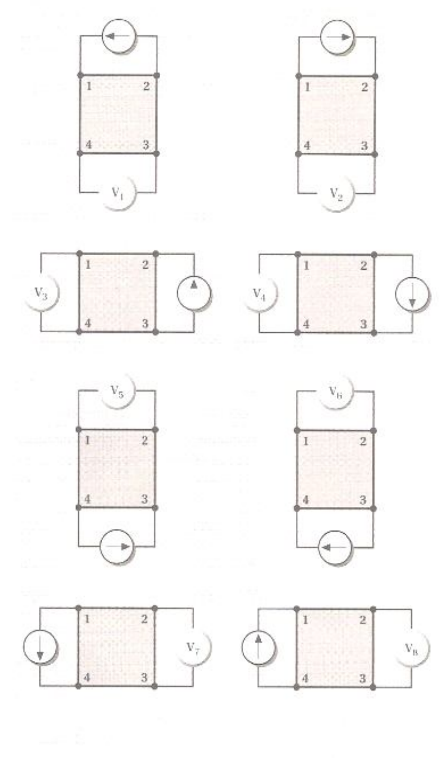

Fig. 2. Van der Pauw technique, derived for sample resistivity measurement.

Two values of resistivty; ρ_A and ρ_B, are then computed as follows:

– (2a)

– (2b)

where: ρ_A and ρ_B are resistivities in ohm-cm

t_s is the sample thickness in cm

V_1-V_8 represent the voltage measurements by the voltmeter

I is the current through the sample in amperes,

f_A and f_B are geometrical factors based on sample symmetry, and are related to the two resistance ratios Q_A and Q_B as shown in the following equations

(3a)

(3b)

Q and *f* are related as follows:

(4)

It is noted that f_A and f_B =1 for perfect symmetry. It is assumed that if ρ_A and ρ_B are not within 10% of one another, the sample is not sufficiently uniform to determine resistivity accurately. Finally, the average of ρ_A and ρ_B is used.

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2.3 Geometrical and Shape Considerations

It has been assumed that to apply Van der Pauw technique, the sample must be flat, continuous, homogenous sample with the probe contacts placed uniformly on the sample perimeter, as depicted in **Fig. 3**. Different computations are carried on volume and surface resistivity measurements, as well as for conductors and semiconductors, [8], [9]

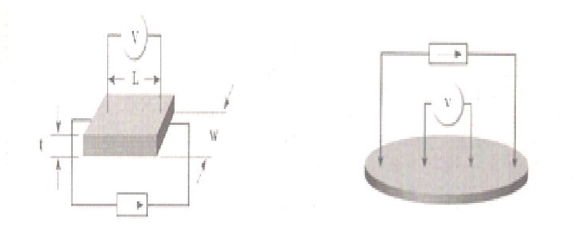

Fig. 3. Volume and surface resistivity measurements using four contacts

2.4 Four Contacts

In this method four arbitrary points or contacts are applied on the sample, in which a constant current source across one pair of contacts is forced through the sample, and the voltage drop is measured on a second pair, as shown in **Fig. 4**. This is referred as constant current method. It is also possible to apply constant voltage source on one pair and measure the current on another, which is termed as constant voltage method. The process is repeated 8 times in Van der Pauw technique in a special convection, to get an average resistivity value**.**

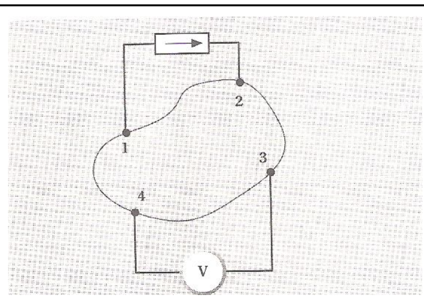

Fig. 4. Four contacts measurement of an arbitrary sample.

2.5 More-Than-Four Contacts

Referring to (2), (3) & (4), it is apparent that V_2 and V_4 measurements are accumulated together for a certain shape constant f_A, as if current source I was injected from contact points 1 and 3, whereas V_1 and V_3 measurements are taken with opposite polarities to remove thermal EMFs effects. The same justification is applied for V_5, V_6, V_7 and V_8 for another shape constant f_B. By inspection, more-than-four contacts can be used with algorithmic computations based on the same method.

Instead of sourcing currents into alternate pairs of contacts, one contact is assumed to be a common point for all current sources injected through the other contacts. Voltage drops across contacts other than the injected current ones, are then measured, as illustrated in **Fig. 5**, in which the square-shaped sample is converted into a triangle-shaped one, with a common point contact.

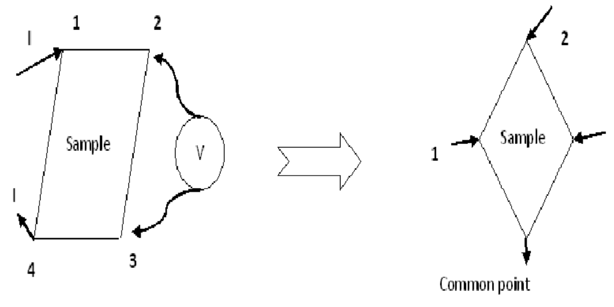

Fig. 5. Converting a four-point contacts sample arrangement to three-point contacts.

In this arrangement, there will be 6 measurements instead of 8, with three geometric factors f_A, f_B and f_C. V_2, V_4 and V_6 are measured as shown in the figure with V_1, V_3 and V_5 are measured with the corresponding currents injected in opposite direction. Van der Pauw equations will be modified as follows:

$$\rho_A = \pi f_A t_S (V_2 - V_1) / 2I \ln 2 \quad (5a)$$

$$\rho_B = \pi f_B t_S (V_4 - V_3) / 2I \ln 2 \quad (5b)$$

$$\rho_C = \pi f_C t_S (V_6 - V_5) / 2I \ln 2 \quad (5c)$$

with Q values:

$$Q_A = (V_6 - V_5) / (V_4 - V_3) \quad (6a)$$

$$Q_B = (V_2 - V_1) / (V_6 - V_5) \quad (6b)$$

$$Q_C = (V_4 - V_3) / (V_1 - V_1) \quad (6c)$$

The average resistivity value is ***($\rho_A + \rho_B + \rho_C$)/3***

This arrangement can be extended to more-than-four contacts, thus generalizing Van der Pauw technique to any number of contacts measurements. The more contacts are selected, the more accurate calculations can be reached, due to the elimination of thermal voltage and temperature coefficients, and also improving the averaging process. In general, there are N equations needed for N-point contacts measurement.

As an alternative approach; a number of four-point measurements are conducted on different contacts of an N-contact sample. The final resistivity value is the mathematical average of all computations carried on these measurements. As an example, consider **Fig. 6**, which shows the arrangements of sample resistivity measurements for different number of contacts. In **Fig. 6a**, currents are sourced through contacts 1, 2, 3, and 4 and voltage drops across other contacts are measured. So, it can be stated that with I injected in contact 1 and the common point, V_{23} and V_{34} are measured. Similarly, when I injected through contact 2, V_{34} and V_{41} are measured. Also, for I injected in 3, V_{12} and V_{24} are measured, and finally for I injected in 4, V_{12} and V_{23} are measured. This is in parallel with four-point measurement of Fig. **6b**, where for example, with I injected in 1 and V_{23} is measured, and when injected in 2, V_{13} is measured, and also with I injected in 3, V_{12} is measured. Finally, current sources are reversed and the measurements are repeated.

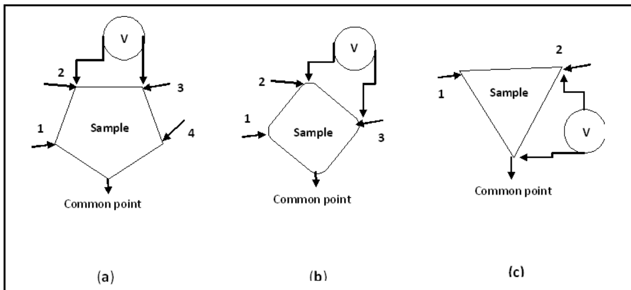

Fig. 6. Different contact arrangement; five-point (a), four-point (b), three-point (c), where current is injected through contacts 1, 2, 3 or 4 and voltage is measured across opposite contacts.

When three contacts are selected, as shown in **Fig. 6c**, the above equations formulation can still hold; with currents injected into one contact and a common point contact, and the voltage is measured across the third contact and the common. If only two contacts are selected, then the 2-point measurement [2], is implemented at the pair of contacts. It employs ohm's law in applying current through the two points, and measuring the voltage drop across them, then averaging the computations with current direction reversed to cancel thermal induced EMFs.

2.6 Thickness Effects

The sample thickness determines whether volume or bulk resistivity measurement, or surface resistivity measurement is required. To differentiate between surface and bulk resistivity, the current has to pass either on the surface or the bulk of sample material, ***(Fig. 3)***. In the former, resistivity is calculated as:

$$\rho = (K_V / t)\ V/I \tag{7}$$

where ρ = volume resistivity in ohm-cm
K_V = test cell constant for volume resistivity based on cell geometry in cm^2
t = sample thickness in cm

And in the latter, it is calculated as

$$\sigma = K_S\ V/I \tag{8}$$

where K_S = test cell constant for surface resistivity based on cell geometry.

3 Measurement Techniques

3.1 Arbitrary Shape

Due to arbitrary shape geometry of a sample, Van der Pauw technique can be applied on each contact pair for the computation of the different geometry factors, *f's*; based on sample symmetry (*f*=1 for perfect symmetry). It is assumed that, for proper formulations, these geometric factors should be within 10% to each other, and therefore contacts positions are selected accordingly. As stated, the more contacts points selected for measurement, the more accurate is the calculation.

3.2 Uniform Shape

For uniform shape geometries, the geometry factor ρ used in Van der Pauw method is the same for any contacts measurement, and so any two pairs of contacts can be used for the computation, and accordingly, only one set of equations is used as

$$\rho = (\pi f t_S / \ln 2)\ V\ /\ I \tag{9}$$

3.3 Three-Dimensional Shape (3-D Van der Pauw)

As an alternative approach for the calculation of bulk or volume resistivity of an arbitrary three-dimensional shape object, the sample can be sliced into a number of equally spaced shapes, each having a uniform geometry factor ρ, in which (9) is applied. Implementation of any number-of-contacts computation algorithms, whether four or more, may be applied on each slice, and then mathematically averaged. Each differential element is treated as a separate geometry with equal uniform resistivity. It is tobe noted that for this arrangement, any point contact is also sliced in the same way.

One application of this treatment, is earth resistivity measurement, in whicht the resistivity of bulk earth volume can be computed by considering it as a uniform surface with measurements applied on any location contacts pairs. The process can be applied on nearby contacts at different distances, and then averaging all computations for a final result.

4 Computerized Measurement Techniques

4.1 Multi-Point Measurement

It can be noticed that computerized calculations are needed for general shape geometries, due to numerous computations applied. With the wide usage of embedded microcomputers and microcontrollers, it is more efficient to employ more generalized measurement techniques.

Random distribution of contact pairs are located on the periphery of any generalized shape geometry, where constant current sources are applied and voltage drops are measured. Due to the elaborated measuring process, a digital form of measurements is needed.

4.2 ADC-DAC Technique

In this technique [10], [11], [12], the analog current source is selected and de-multiplexed to 2^N contact points for sourcing currents, using $1\text{-}2^N$ De-Multiplexer (de-MUX). N selectors are used for the selections. The corresponding voltages, in general, measured at 2^M digital contact points, are consequently selected to one value with the help of $2^M\text{-}1$ Multiplexer (MUX), and M selectors, as depicted in **Fig. 7**. Normaly $N=M$.

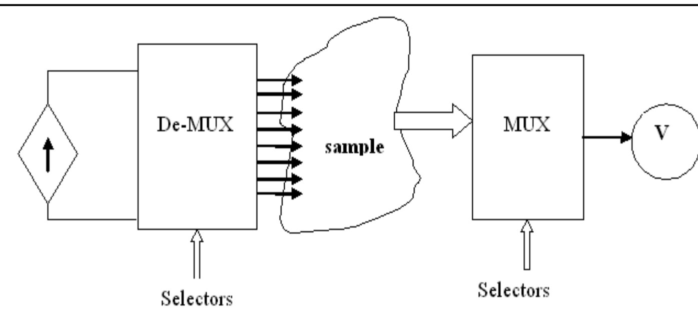

Fig. 7. The use of multiplexers and de-multiplexers in multi-point contacts measurements is implemented for digitized computations.

As an example, 1-8 de-MUX is used with 3 selectors to direct the sourced current to one of the 8 point contacts, and the voltage at any other point contact is selected with a *normaly* 8-1 MUX and 3 selectors.

4.3 Microcontrollers

With the use of embedded computers [13], such as microcontrollers, the functions of MUX and de-MUX, for multi-point measurement technique, can be conducted with ease. All required calculations are then computed in the embedded processor with the help of a source program written in Basic, C or Assembly. Figure 8 depicts the use of one or more ports of microcontroller for the calculation of resistivity of any general shape geometry

Currents are sourced from one outlet port; port A, and then redirected to one point contact using a 1-8 de-MUX, and the voltage at any other point contact is selected with a 8-1 MUX. The voltage is converted to a digital value with an analog-to-digital (ADC), before inputed in the inlet port; port B. All computations are carried out in the source program efficiently. The selectors, themselves, are outputted from the microcontroller ports. It is noticesd that only one port can be used for the measurements through the source program.

The procedure can be repeated and the resistivity result averaged for computation improvement, using the microcontroller timer or oscillator.

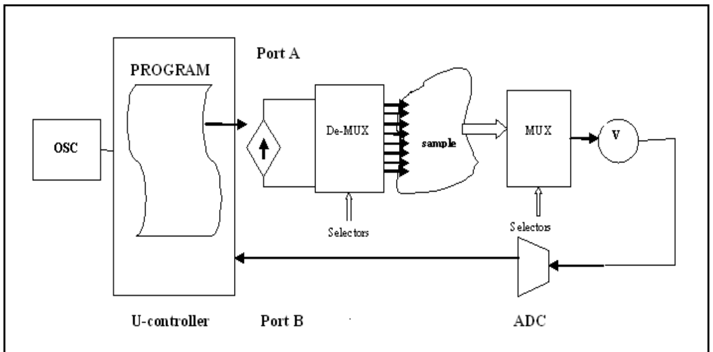

Fig. 8. Microcontroller implementation of resistivity measurement.

5 CONCLUSION

The Van dr Pauw technique for bulk and surface resistivity measurement has been expanded for different geometric shapes, such as arbitrary 3-D shape, both for conductors and high resistance material. Computational algorithms are generalized for any number of contacts measurements, such as more-than-four and less-than-four point contacts. The more points contacts used, the more accurate is the computation for an arbitrary shaped geometry. This procedure is therefore suitable for larger samples applications such as earth resistivity measurement.

1-2^N de-multiplexer and 2^N-1 multiplexer are used to practically implement for this technique; for example 1-8 de-MUX and 8-1 MUX. Embedded computers, such as microcontrollers, can simplify the measurements procedure by using input and output ports, controlled by a source program written in Basic, C or Assembly.

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